Econ 2125 Optimisation for Economics and Financial Economics *

Week 5 Tutorial Questions

Question 1 Prove that a closed subset of a compact set in $\mathbb{R}^m$ is compact.

Question 2
(a) Prove that the intersection of compact sets is compact.

(b) Prove that the finite union of compact sets is compact.

(c) Is the following statement true or false? If it is true, prove it. If it is false, present a counter example:
Statement: *An infinite union of compact sets must be compact.*

Question 3: Draw a level curve for each of the following functions:

(a) $f(x, y) = y - 2x$

(b) $f(x, y) = y/x$

(c) $f(x, y) = y - x^2$

Question 4: Write the following linear functions in matrix form:
(a) $f(x, y, z) = 2x - 3y + 5z$

(b) $f(x, y) = (2x - 3y, x - 4y, x)$

(c) $f(x, y, z) = (x - z, 2x + 3y - 6z, x + 2y)$

Question 5: Write the following quadratic forms in matrix form:
(a) $x^2 - 2xy + y^2$

(b) $5x^2 - 10xy - y^2$

(c) $x^2 + 2y^2 + 3z^2 + 4xy - 6xz + 8yz$

Question 6: Write two functions from $\mathbb{R}$ to $\mathbb{R}$ that are not polynomials.

Question 7

Consider the function $f : \mathbb{R} \to \mathbb{R}$ with

$$f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 2x - 1 & \text{if } x \notin \mathbb{Q} \end{cases}$$

Is f continuous at $x = 1$? Explain.

Question 8: For each of the following functions, what is its domain and range? Is it one-to-one? If it is one-to-one, write the expression for the inverse function. Is it onto?

(a) $f(x) = 3x - 7$

(b) $f(x) = e^x$

(c) $f(x) = \sqrt{x - 1}$

(d) $f(x) = x^2 - 1$