Assignment-3

ANU, ECON2125/6012, Semester-1 2023

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Question-1 Consider the following problems (18 marks)

(a) $max_{x,y}\ x^2+y^2$ subject to $r^2 \le 2x^2+6y^2 \le s^2$ with $0<r<s$

(b) $\min_{x,y}\ x^2+y^2$ subject to $r^2 \le 2x^2+6y^2 \le s^2$ with $0<r<s$

(i) Solve problem *(a)*

(ii) Solve problem *(b)*

(iii) How much does the optimal value of the function change if s changes by .1 unit in problem (a). How much does the optimal value of the function change if r changes by .1 unit in problem (a).

(iv) Check the second order condition for problem (b).

(v) What are the geometric interpretations of (a) and (b)?

Question-2 Find the solution to (10 marks)

$$\min_{\mathbf{x}} -\sum_{i=1}^{N} \log(\alpha_i + x_i) \text{ subject to } x_i \ge 0 \text{ and } \sum_{i=1}^{N} x_i = 1 \text{ with } \alpha_i > 0$$

Is the objective function $-\sum_{i=1}^{N} \log(\alpha_i + x_i)$ concave or convex? Prove your answer.

Question-3 (10 marks)

Suppose a consumer has a wealth of W. There is a probability p of a loss of L if an adverse event happens. The consumer can buy insurance that will pay him Q in case that the loss happens. The consumer has to pay π per dollar insured as the premium. The consumer's problem can be formulated as

$$\max_Q\ pU(W-L-\pi Q+Q)+(1-p)U(W-\pi Q)$$

i) Find the first order condition.

ii) Note that the expected profit for the insurance company is $(1-p)\pi Q - p(1-\pi)Q$. Suppose that the market is competitive which forces the expected profit to be zero. In this case, find π.

iii) If the consumer is strictly risk-averse i.e. $d^2U/dW^2 < 0$, show that under (ii) the consumer fully insure against the lost i.e. $Q^* = L$

Question-4 (12 marks)

An investor must choose a portfolio $\mathbf{x} = (x_1, ..., x_n)^T$ where x_j is the proportion of assets invested in j-th security. The return to the security is $M = \boldsymbol{\mu}\mathbf{x} = \sum_{j=1}^{n} \mu_j x_j$ where $\boldsymbol{\mu}$ is the vector containing mean returns to each security. The risk on the portfolio is measured by the variance of returns $V = \mathbf{x}^T \boldsymbol{\Sigma} \mathbf{x} = \sum_{j=1}^{n} \sum_{k=1}^{n} \sigma_{jk} x_j x_k$ where $\boldsymbol{\Sigma}$ is the variance-covariance matrix of security returns. A portfolio is efficient if there is no other portfolio with either a higher return and lower risk or with a lower risk at the same level of return.

1. For the problem of

$$\max_x M(\mathbf{x}) \text{ subject to } \mathrm{V}(\mathbf{x}) \leq V_0, \mathbf{x} \geq \mathbf{0}, \mathbf{i}^T \mathbf{x} = 1$$

find the first order conditions and show the solution yields an efficient portfolio.

2. For the problem of

$$\min_x V(\mathbf{x}) \text{ subjec to } \mathrm{M}(\mathbf{x}) \geq M_0, \mathbf{x} \geq \mathbf{0}, \mathbf{i}^T \mathbf{x} = 1$$

find the first order conditions and show the solution yields an efficient portfolio.