The Dynamics of Betrand Competition with Leapfrogging Investments

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Overview of results

Dynamic game in discrete time, infinite horizon

- Standard Bertrand price competition with two firms
- Cost reducing investment decisions by firms
  - Exogenous technological progress
  - Discrete decision to acquire state of the art technology

We show

- This simple, straightforward extension of the classic static model of Bertrand price competition results in surprisingly rich, complex, and even counter-intuitive equilibrium outcomes.
- How to compute *all* Markov perfect equilibria of this dynamic game

The Bertrand Investment Paradox

Why should Bertrand competitors undertake cost-reducing investments?

- Suppose a pair of duopolists *simultaneously* invest in the state of the art low cost production technology with marginal cost c
- Bertrand price competition following these investments will lead to a price of $p = c$ and *zero profits for each firm*
- If each firm earns zero profits *ex post*, why would either have incentive to invest *ex ante*?

If duopolists can coordinate their actions, they will be able to avoid "bad" outcome of simultaneous investment

Resolution to Bertrand Investment Paradox

Earlier work:

- Frudenberg et al. (1983 RIE), Reinganum (1985 QJE), Frudenberg and Tirole (1985 ReStud), ...
- Riordan and Salant (1994 JIE): Preemption and rent dissipation (unique equilibrium)

We show:

- Many types of endog. coordination is possible in equilibrium
  - Leapfrogging (alternating investments)
  - Preemption (investment by cost leader)
  - Duplicative (simulatnaoues investments)
- The equilibria are generally inefficient due to overinvestment
  - Duplicative or excessively frequent investments

A new interpretation of price wars

We show:

- Price path is piecewise flat and non-increasing
- Price wars occur when the high cost firm leapfrogs its rival to become the new low-cost leader
- These price wars lead to permanent price declines, unlike the conventional interpretation of price wars as punishments for periodic breakdowns in tacit collusion

We compute ALL Markov perfect equilibria

New solution approach consisting of:

1. State recursion: for finding *any stage equilibrium*
2. Recursive Lexicographic Search (RLS): for finding *all MPE*

Traditional solution approach fails

- Value function iterations (*time recursion*) fails due to multiplicity of equilibria
  - Not a contraction mapping, convergence is not guaranteed
  - If convergence is achieved, only a single equilibrium is found
  - Bellman operator induces equilibrium selection rule
- Existing "All solution" homotopy methods: fails due to too many bifurcations along the path
- Danger of imposing symmetry: *most MPE are asymmetric* in this game

Behavioral model

Basic Setup

- Discrete time, infinite horizon ($t = 1, 2, 3, \ldots$)
- Two firms, homogenous/differentialed goods, no entry or exit
- Each firm maximizes expected discounted profits

Firms face two decisions

- Set product prices (simultaneously)
- Whether to invest in state of the art production technology
  1. *Simultaneous moves*
  2. *Alternating moves*: The right to move follows a Markov process (deterministic as a special case)

Behavioral model - cont.

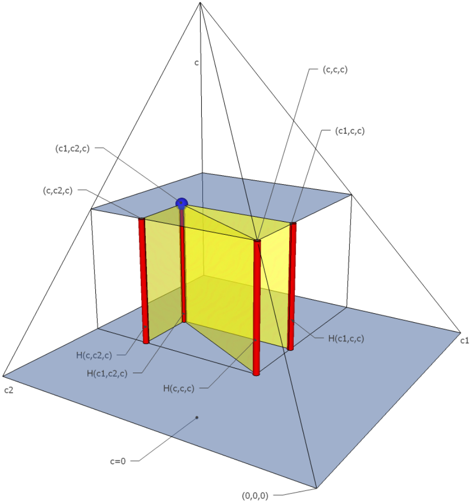
Within period returns

- Prices determined in *static* Bertrand-Nash game
- Duopolists profits increase in cost advantage

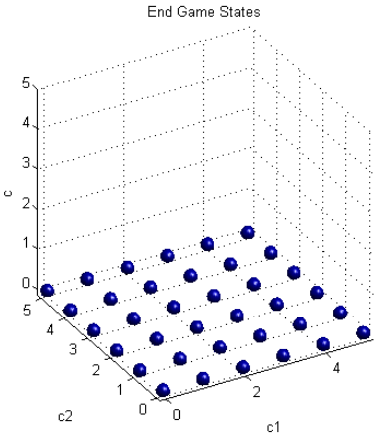
State of the art technology

- Pay investment cost of $K(c)$ to obtain marginal cost $c \geq 0$
- Additive shocks to investment cost/benefits (private information)
- Time to build: state of the art technology is operational after a one period lag
- State of the art costs follows exogenous Markov process

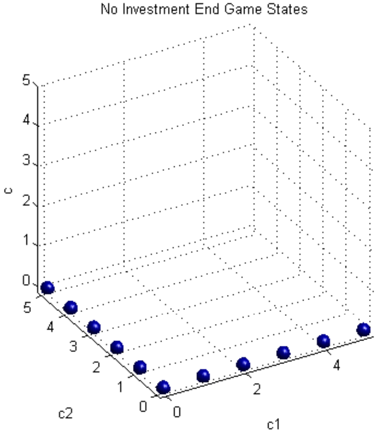
State space of the game: a “quarter pyramid”
$S = \{(c_1, c_2, c) | c_1 \geq c,\ c_2 \geq c, c \in [0, \overline{c}]\}$



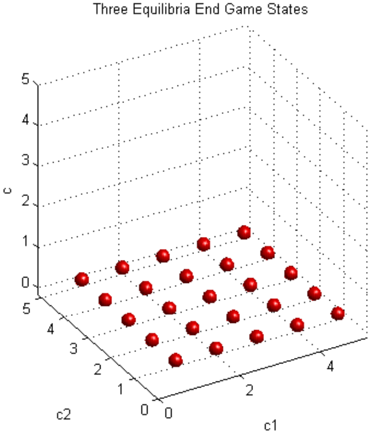
End Games States



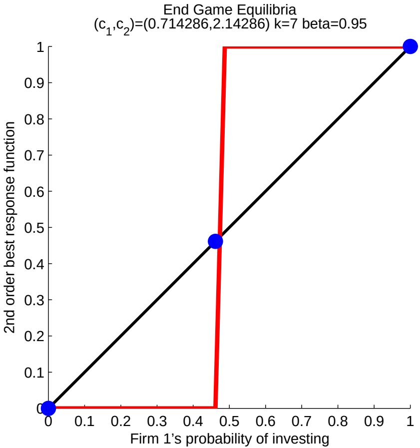
No Investment End Games States



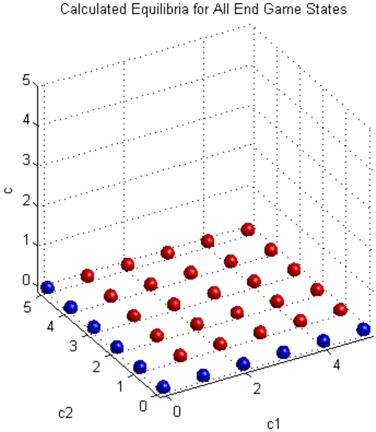
Multiple Equilibria End Games States



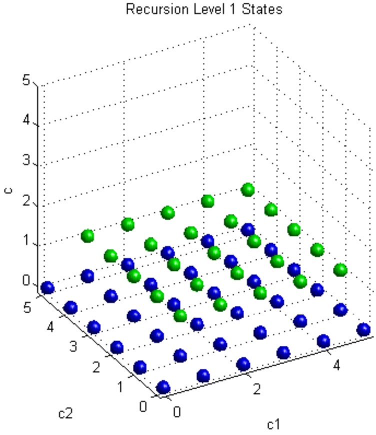
Second order best response function, $\eta = 0$



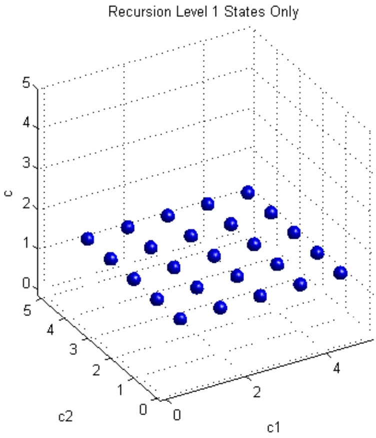
Calculated Equilibria for End Games States



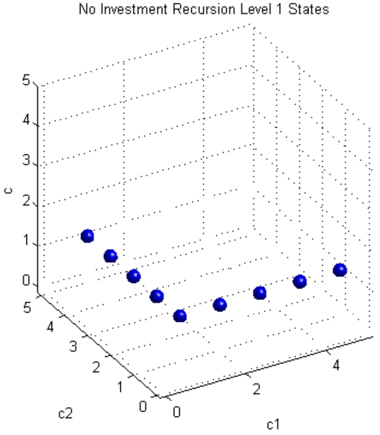
Recursion Level 1 States



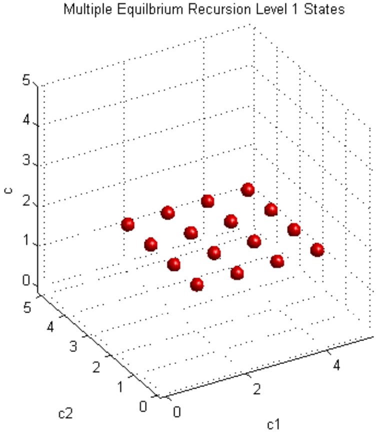
Recursion Level 1 States, isolated



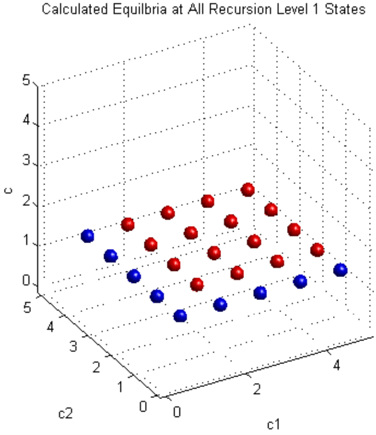
No Investment Recursion Level 1 States



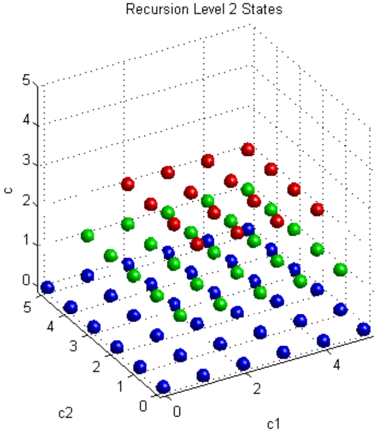
Multiple Equilibria Recursion Level 1 States



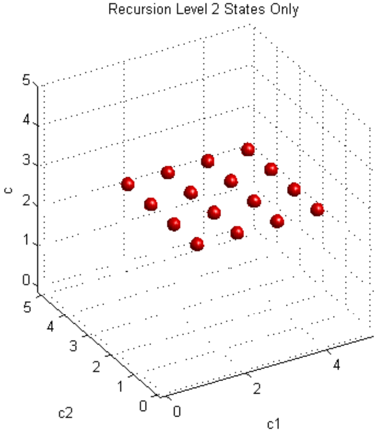
Calculated Equilibria Recursion Level 1 States



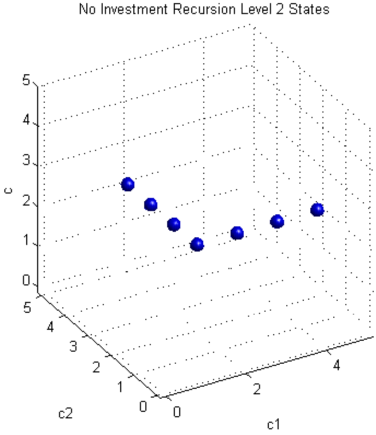
Recursion Level 2 States



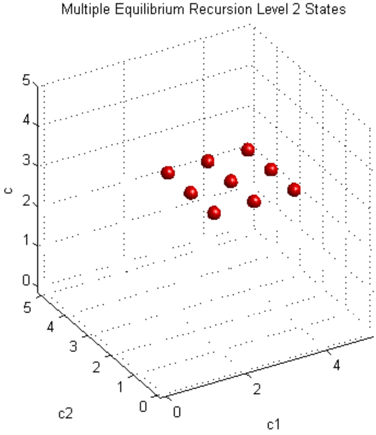
Recursion Level 2 States, isolated



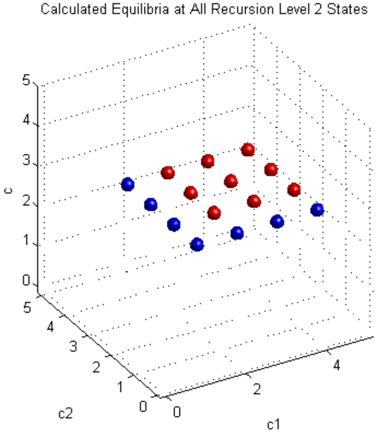
No Investment Recursion Level 2 States



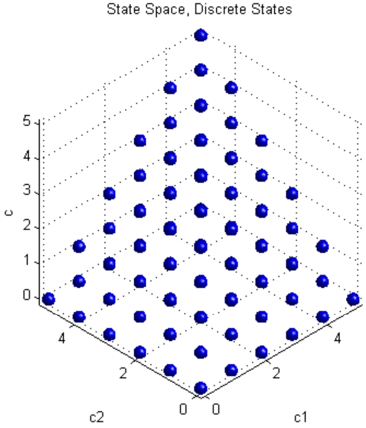
Multiple Equilibria Recursion Level 2 States



Calculated Equilibria Recursion Level 2 States



Continue recursion to calculate equilibria in all states



Recursive Lexicographic Search Algorithm

Building blocks of RLS algorithm:

1. State recursion algorithm solves the game *conditional on* equilibrium selection rule (ESR)
2. RLS algorithm efficiently cycles through *all feasible* ESRs

Properties of RLS algorithm:

- *Complete*: Computes all MPE equilibria of the game
- *Fast*: time spent of search of feasible ESRs is negligible in comparison to time spent on solving the game
  - Efficiently skip infeasible ESRs
  - Re-use results of previously computed subgames

Represent ESR as *equilibrium string* of digits

Use numbers in base-K number system with digits $0, 1, .., K-1$

Dependence preserving property:

Any point of the state space may depend on points to the left and not the points to the right

corner

edges

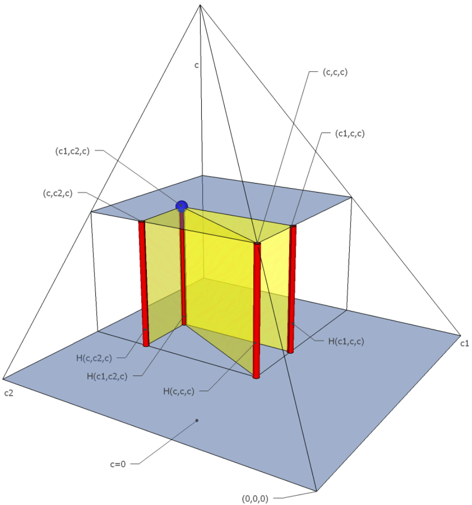
interior

	c	e	e	e	e	i	i	i	i	c	e	e	i	c
ESR string	14	13	12	11	10	9	8	7	6	5	4	3	2	1
c	0	0	0	0	0	0	0	0	0	1	1	1	1	2
$c1$	0	0	0	2	1	2	2	1	1	1	1	2	2	2
$c2$	0	2	1	0	0	2	1	2	1	1	2	1	2	2

End game

State space of the game: a "quarter pyramid"

$S = \{(c_1, c_2, c) | c_1 \geq c,\ c_2 \geq c, c \in [0, \bar{c}]\}$



Particular ESRs examples

ESR string	c	e	e	e	e	i	i	i	i	c	e	e	i	c
	14	13	12	11	10	9	8	7	6	5	4	3	2	1
c	0	0	0	0	0	0	0	0	0	1	1	1	1	2
c1	0	0	0	2	1	2	2	1	1	1	1	2	2	2
c2	0	2	1	0	0	2	1	2	1	1	2	1	2	2

End game

Examples:

0	0	0	0	0	0	0	0	0	0	0	0	0	0	First equilibrium always
	0	0	2	2		2	0			0	2			High cost to invest
	2	2	0	0		0	2			2	0			Low cost to invest
1					1			1	1			1	1	Mixed when equal

All possible ESRs

Lexicographic order

	c	e	e	e	e	i	i	i	i	c	e	e	i	c
ESR string	14	13	12	11	10	9	8	7	6	5	4	3	2	1
Lexicograph	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0	2
	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	1	1
	0	0	0	0	0	0	0	0	0	0	0	0	1	2
	0	0	0	0	0	0	0	0	0	0	0	0	2	0
	0	0	0	0	0	0	0	0	0	0	0	0	2	1
	0	0	0	0	0	0	0	0	0	0	0	0	2	2
	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0	1
														…
	2	2	2	2	2	2	2	2	2	2	2	2	2	0
	2	2	2	2	2	2	2	2	2	2	2	2	2	1
	2	2	2	2	2	2	2	2	2	2	2	2	2	2

4,782,969

Recalculation of fesibility condition for new ESR

Avoid recalculation of subgames

	c	e	e	e	e	i	i	i	i	c	e	e	i	c	
ESR string	14	13	12	11	10	9	8	7	6	5	4	3	2	1	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	always admissible
Nr of eqb	1	1	1	1	1	3	3	3	3	1	1	1	**3**	1	
	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	
	0	0	0	0	0	0	0	0	0	0	0	0	0	**2**	
	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0	admissible, solve
	1	1	1	1	1	3	3	3	3	1	1	1	3	*	

No changes in the solution of the game including the number of stage equilibria

Might have changed

Jumping over blocks of infeasibles ESRs

Using block structure of lexicographic ordering

	c	e	e	e	e	i	i	i	i	c	e	e	i	c	
ESR string	14	13	12	11	10	9	8	7	6	5	4	3	2	1	Iteration:
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	1
	1	1	1	1	1	3	3	3	3	1	1	1	**3**	**1**	1a
	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	
	0	0	0	0	0	0	0	0	0	0	0	0	0	2	
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	2
	1	1	1	1	1	3	3	3	3	1	1	1	**3**	**1**	2a
	0	0	0	0	0	0	0	0	0	0	0	0	1	**1**	
	0	0	0	0	0	0	0	0	0	0	0	0	1	2	
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**2**	**0**	3
	1	1	1	1	1	3	3	3	**3**	**1**	**1**	**1**	**3**	**1**	
	0	0	0	0	0	0	0	0	0	0	0	0	2	**1**	3a
	0	0	0	0	0	0	0	0	0	0	0	0	2	2	
	0	0	0	0	0	0	0	0	0	0	0	**1**	0	0	3b
														…	
	0	0	0	0	0	0	0	0	0	0	**1**	0	0	0	3c
														…	
	0	0	0	0	0	0	0	0	0	**1**	0	0	0	0	3d
														…	
	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	4
														…	

RLS algorithm: running times

$K = 3$

Simultaneous moves	$n = 3$	$n = 4$
Total number ESRs	4 782 969	3 948 865 611
Number of feasible ESRs	127	46 707
Time used	0.008 sec.	0.334 sec.

Simultaneous moves	$n = 5$
Total number ESRs	174449211009120166087753728
Number of feasible ESRs	192 736 405
Time used	45 min.

Alternating moves	$n = 5$
Total number ESRs	174449211009120166087753728
Number of feasible ESRs	1
Time used	0.006 sec.

Road Map for the rest of Talk

Results and Simulations

1. Resolution to the Bertrand investment paradox
2. Sufficient conditions for uniqueness of equilibria
3. Characterization of the set of equilibrium payoffs
4. Efficiency of equilibria
5. Leap-frogging or preemption and rent-dissipation

Resolution to the Bertrand investment paradox

Theorem (Solution to Bertrand investment paradox)

If investment is socially optimal at a state point $(c_1, c_2, c) \in S$, then

- *no investment by both firms cannot be an MPE outcome in the subgame starting from (c_1, c_2, c) in either the simultaneous or alternating move versions of the dynamic game.*

Multiplicity of equilibria

Theorem (Sufficient conditions for uniqueness)

In the dynamic Bertrand investment and pricing game a sufficient condition for the MPE to be unique is that

1. *firms move in alternating fashion (i.e. $m \neq 0$), and,*
2. *for each $c > 0$ in the support of π we have $\pi(c|c) = 0$.*

1. Corollary: If firms move simultaneously, equilibrium is generally *not unique*.
2. Corollary: If technological change is stochastic, equilibrium is generally *not unique*.

Multiplicity of equilibria

Theorem (Number of equilibria in simultaneous move game)

If investment is socially optimal, and the support of the Markov process $\{c_t\}$ for the state of the art marginal costs is the full interval $[0, c_0]$ (i.e. continuous state version),

- *the simultaneous move Bertrand investment and pricing game has a continuum of MPE.*

Pay-offs in the simultaneous move game

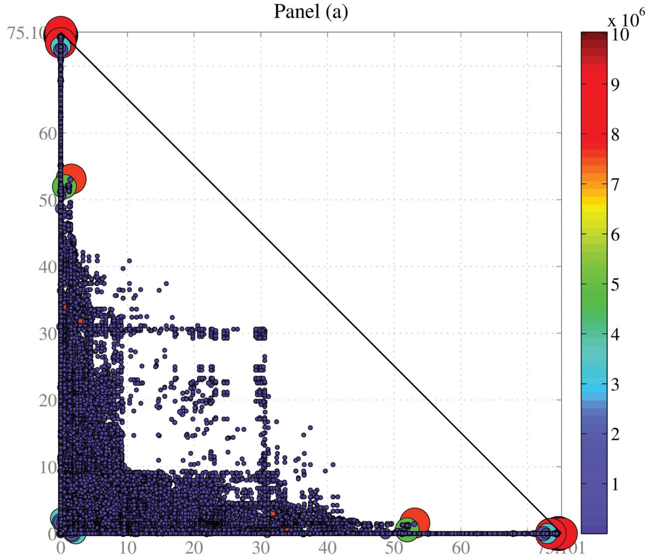
Theorem (Triangular payoffs in the simultaneous move game)

Suppose that the $\{c_t\}$ process has finite support, that there are no idiosyncratic shocks to investment (i.e. $\eta = 0$) and that firms move simultaneously

- *The (convex hull of the) set of the expected discounted equilibrium payoffs at the apex state $(c_0, c_0, c_0) \in S$ is a triangle*
- *The vertices of this triangle are at the points $(0, 0)$, $(0, V_M)$ and $(V_M, 0)$ where $V_M = v_{N,i}(c_0, c_0, c_0)$ is the expected discounted payoff to firm i in the monopoly equilibrium where firm i is the monopolist investor.*

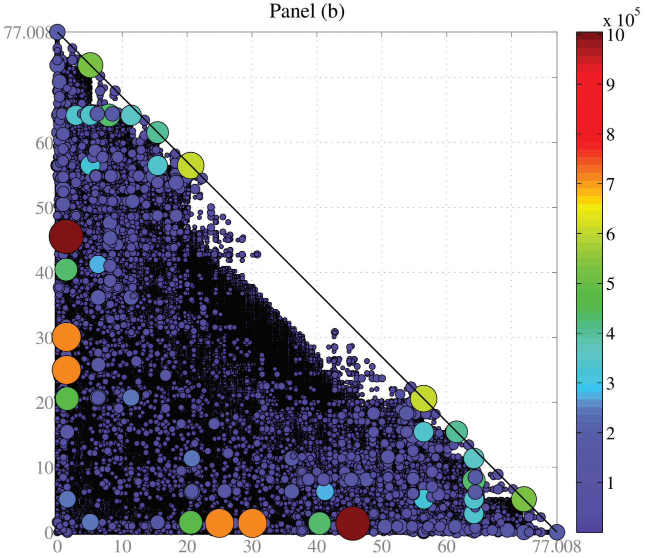
Pay-off map

Deterministic (one step) Technological Progress, Simultaneous moves



Pay-off map

Random Technological Progress, Simultaneous moves

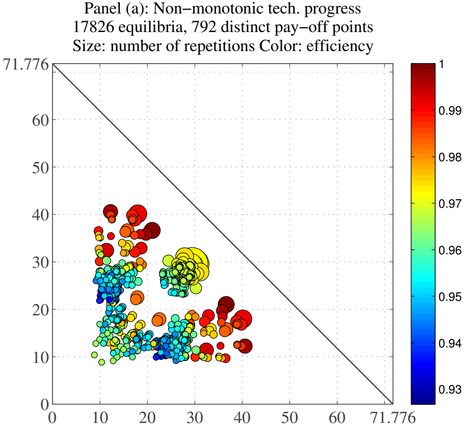


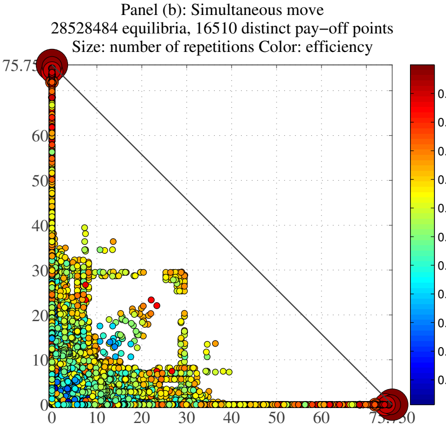
Pay-offs in the alternating move game

Theorem (Equilibrium payoffs in the alternating move game)

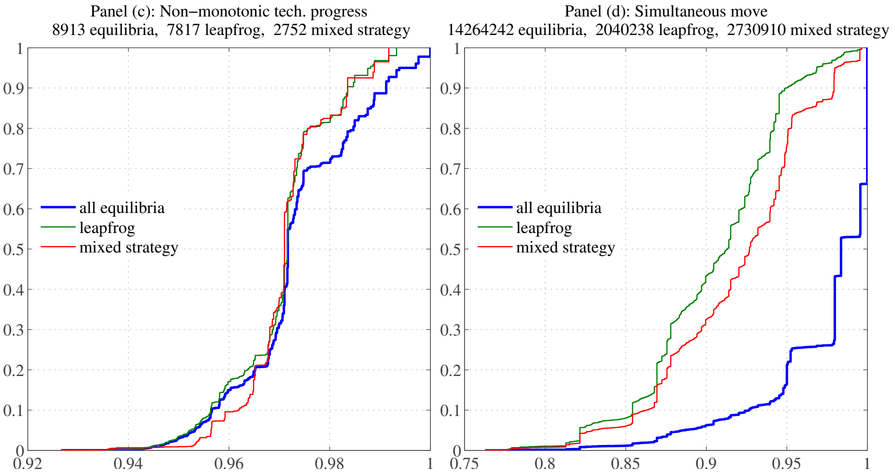
The (convex hull of the) set of expected discounted equilibrium payoffs at the apex state $(c_0, c_0, c_0) \in S$ *of the alternating game is a strict subset of the triangle with the vertices* $(0, 0)$*,* $(0, V_M)$ *and* $(V_M, 0)$

Pay-offs: alternating vs simultaneous move games





Efficiency: alternating vs simultaneous move games



Theorem (Monopoly outcome in simultaneous move game)

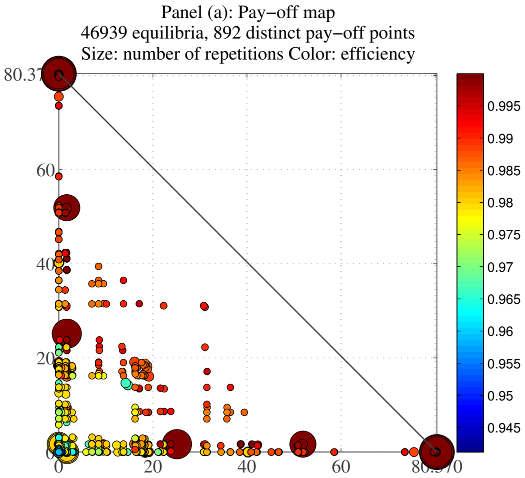
If investment is optimal for the social planner, in the sense that investment costs are not prohibitively high, then there exist two symmetric "monopoly" MPE in the simultaneous move game.

Theorem (Existence of efficient non-monopoly equilibria)

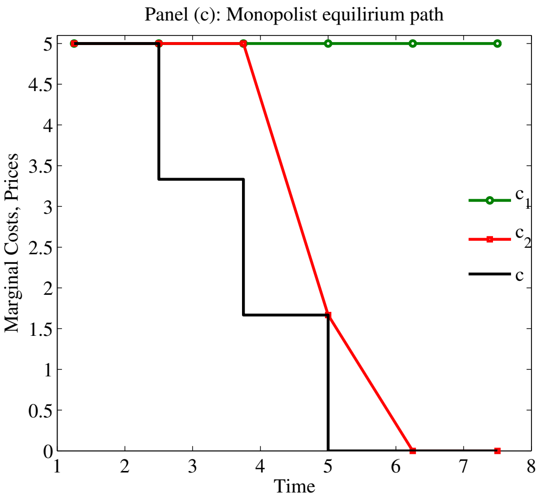
In both the simultaneous and alternating move investment games, there exist fully efficient non-monopoly equilibria.

Efficiency of equilibria

Simultaneous move game

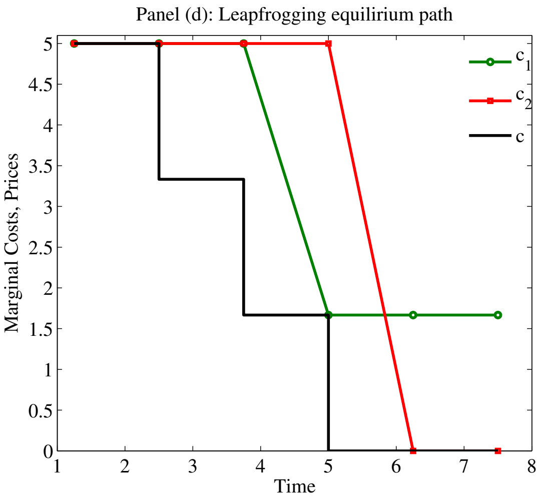


Monopoly outcome
Simultaneous move game



Efficient leapfrogging

Simultaneous move game



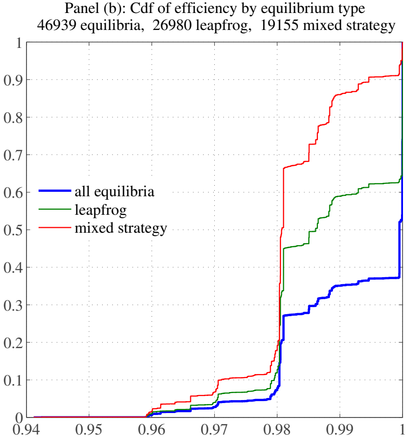
Efficiency of equilibria
Simultaneous move game

Theorem (Inefficiency of mixed strategy equilibria)

A necessary condition for efficiency in the dynamic Bertrand investment and pricing game is that along MPE path only pure strategy stage equilibria are played.

Distribution of efficiency of equilibrium type

Simultaneous move game



NB: Red line jumps up at .99998, so there are no mixed strategy efficient equilibria

Riordan and Salant: Full Preemption

Theorem (Riordan and Salant, 1994)

Consider the continuous time investment game where

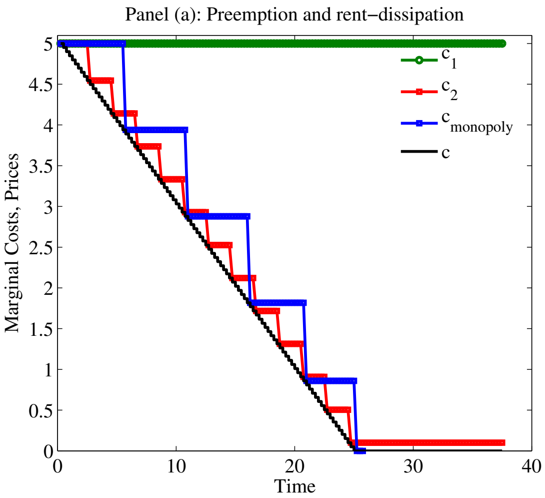
1. *right to move alternates deterministically.*
2. $K(c) = K$ *and is not prohibitively high.*
3. *technological progress is deterministic:* $c(t)$ *is a continuous, non-increasing function*

Then there exists a unique MPE of the continuous time investment game that involve

- *preemptive investments: only one firm and no investment in equilibrium by its opponent.*
- *rent dissipation: discounted payoffs of both firms in equilibrium is* 0*, so the entire surplus is wasted on excessively frequent investments by the preempting firm.*

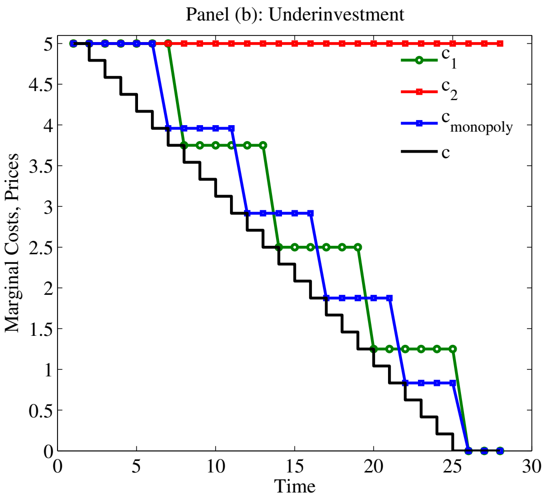
Riordan and Salant: Full preemption and rent dissipation

Confirm the result with high K and small dt



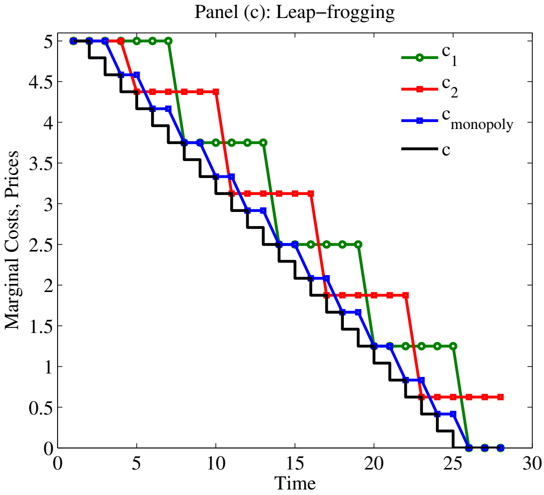
Underinvestment

Rent-dissapation is not a general outcome - disappears when K is low relative dt



Leap-frogging

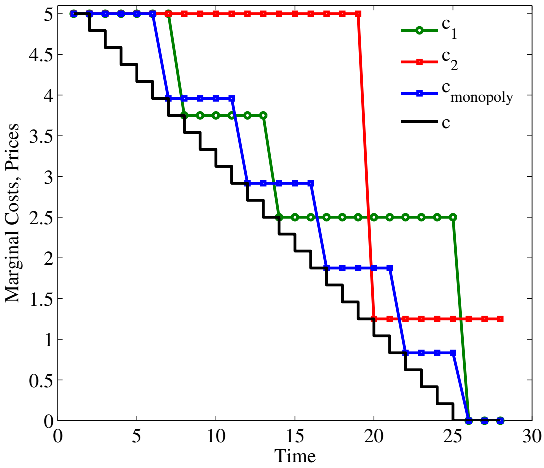
Preemption is not the general outcome - disappears when K is even lower



Random alternation → Leapfrogging

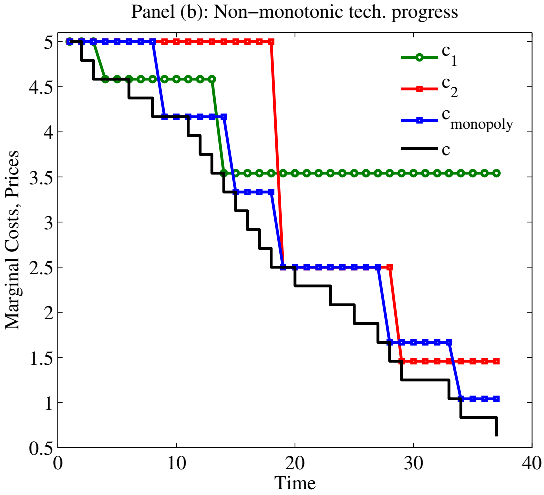
Riodan and Salant's result is not robust

Panel (a): Random alternating moves



Random onestep technology → Leapfrogging

Riodan and Salant's result is not robust



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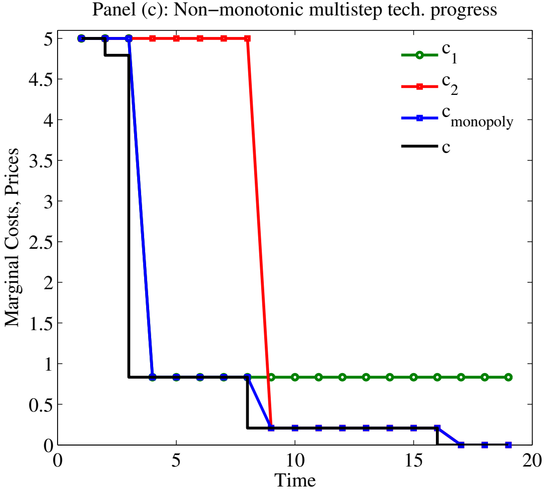
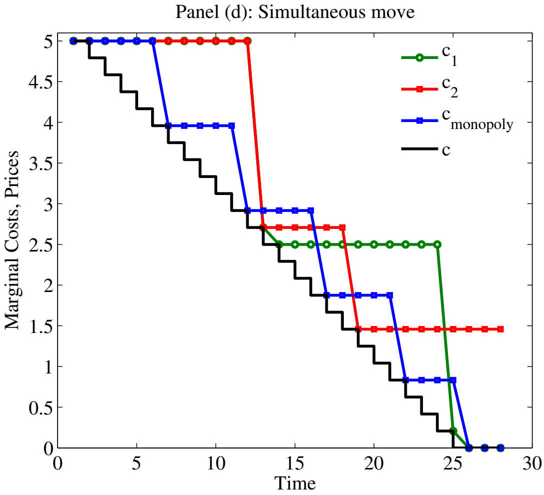
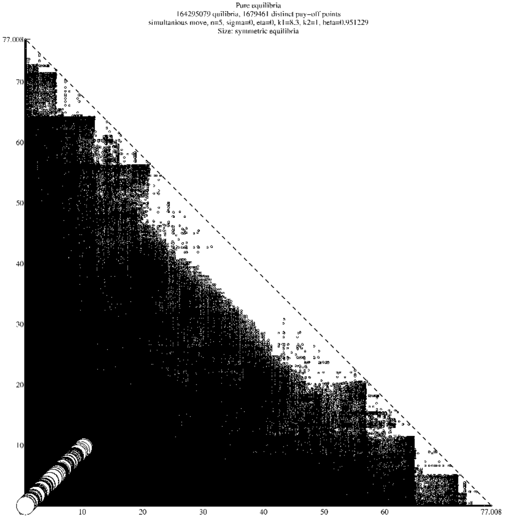


Simultaneous moves: Leapfrogging

Riordan and Salant's conjecture is wrong



Symmetric equilibria: $V_1(c_1, c_2, c) = V_2(c_2, c_1, c)$



Failure of homotopy approach

Homotopy parameter: Variance of idiosyncratic shocks in investment decisions

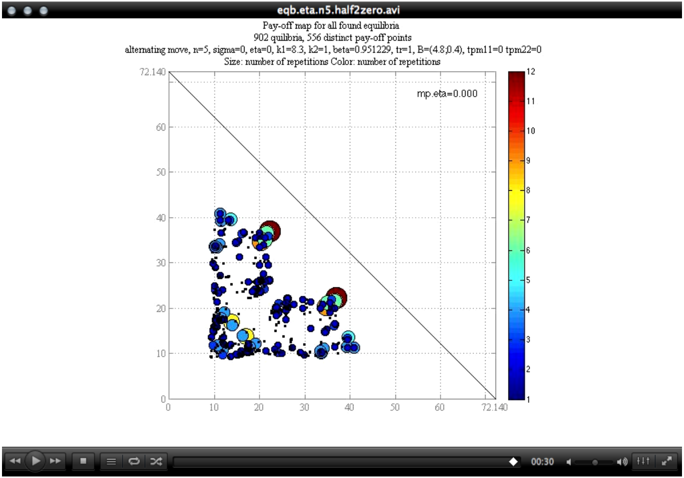
- In each period each firm incurs additive random costs/benefit from not investing and investing
- η is a scaling parameter
- game of incomplete information when $\eta > 0$

Doraszelski and Escobar (2008 ***Theoretical Economics*****)**

- Equilibria in our game are subject to their theory of regular MPE in dynamic stochastic games
- η is the homotopy parameter in the path-following methods
- **Problem:** multiplicity of equilibria $\rightarrow$ too many bifurcations along the path

Failure of homotopy approach

Video: Set of equilibrium outcomes as variance of shocks decreases to zero



Theoretical conclusions

- Endogenous coordination (e.g. leapfrogging) is possible in equilibrium of dynamic Bertrand investment game
- Leapfrogging gives new interpretation of the price wars
- Numerous MPE equilibria and "Folk theorem"-like result
- Most equilibria are inefficient due to overinvestment

Methodological conclusions

- When equilibrium is not unique the computation algorithm inadvertently acts as an *equilibrium selection mechanism*
- State recursion algorithm is preferred to time iterations
- Imposing symmetry restriction on equilibria knocks out most equilibria in the model
- Plethora of Markov perfect equilibria leads to new paradox: How can firms, without any explicit communication, coordinate on a single equilibrium in these games when there is generally such a vast multiplicity of possible equilibria.