Making Waves: the Effect of Macro Shocks on the Auto Market

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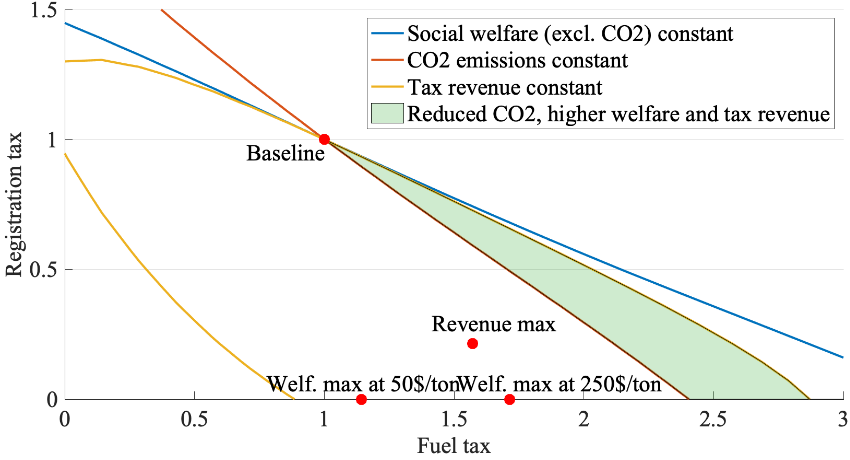
Micro-based disaggregate equilibrium models of the auto market

- Early work on disaggregate micro models of the car market by Chuck Manski and Sherman 1980 using data from Israel
- My late MIT classmate, Jim Berkovec, in his 1983 MIT PhD thesis developed a micro-founded equilibrium of the automobile market, published in RAND 1985 "New Car Sales and Used Car Stocks: A Model of the Automobile Market"
- These were static discrete choice models. In my own 1983 PhD thesis at MIT, "Stationary Equilibrium in a Market for Durable Goods" I attempted to add dynamics: when to keep the current car versus trade it for a new or used one?
- Due to the limitations of my approach (which treating the state variable of a car, odometer, as continuous) I could only model a dynamic equilibrium with zero transactions costs, where people are indifferent between keeping or trading their cars.
- Subsequent work in the 1990s such as Pinelopi Goldberg's Stanford thesis and the famous BLP model in IO reverted to static models of car choice and focused on sales of new cars.

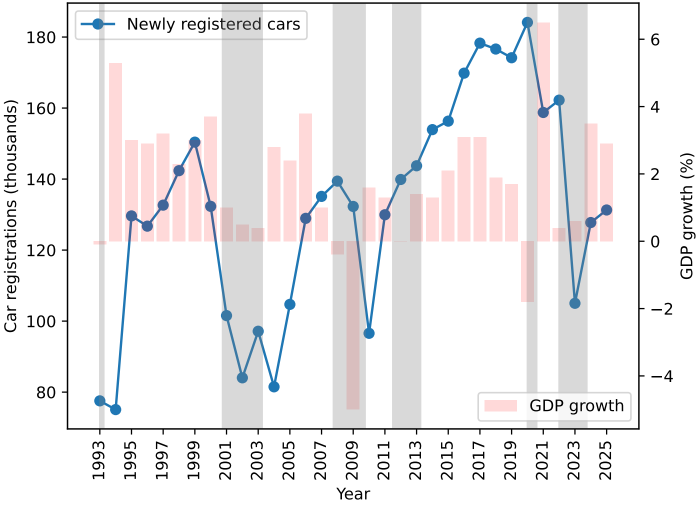
Renewed progress on dynamic models with transactions cost in 2000s

- Dimitri Stolyarov 2002 *JPE* "Turnover of Used Durables in Stationary Equilibrium: Are Older Goods Traded More?"
- Susanna Esteban and Matt Shum 2007 *RAND* "Durable Goods Oligopoly with Secondary Markets: The Case of Automobiles"
- Gavazza, Lizzeri and Roketskiy 2014 *AER* "A Quantitative Analysis of the Used Car Market:"
- Gillingham, Iskhakov, Munk-Nielsen, Rust and Schjerning 2022 *JPE* "Equilibrium Trade in Automobiles"
- latter paper introduced a "triply nested fixed point algorithm" that allowed us to estimate preference parameters for new and used cars using Danish microdata where we did not directly observe used car prices, but could compute them by enforcing stationary equilibrium in the used car market.
- Our "trick" was to treat car "state" as discrete (i.e. car age) and formulate auto trading as a discrete choice problem that could accomodate transactions costs (which we found to be empirically important).
- Using this model we found that Denmark's ultra high 180% new car registration tax put it over the top of the Laffer curve.

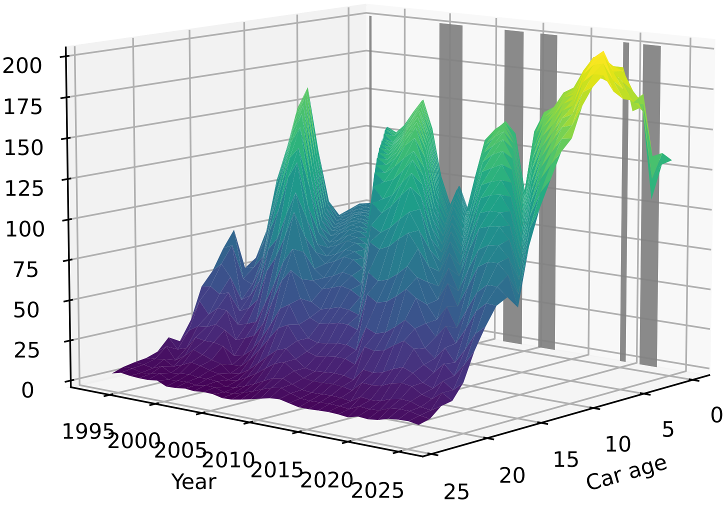
Stationary Equilibrium Counterfactuals for Denmark



Macro cycles affect new car sales in Denmark



Cycles in new car sales produce waves in the stock of cars



Relaxing stationarity: adding macro shocks to auto equilibrium models

- Cooper, Haltiwanger and Power AER 1999, "Machine Replacement and the Business Cycle: Lumps and Bump" were one of the first to try to construct a micro-founded model of shock propagation, but but focusing on replacement investment decisions by firms rather than consumers.
- They showed how endogenous coordination over the timing of machine replacement both propagates and amplifies macroeconomic shocks Forward looking behavior is crucial to their explanation since investment "is more likely to be procyclical the more persistent are shocks and the more important are fixed adjustment costs."
- The age distribution of capital is important for aggregate investment dynamics since firms are more likely to replace older machines, resulting in "replacement cycles" that form when firms independently choose to replace older machines a start of a boom, resulting in spikes in investment that turn into waves of vintage capital that are more likely to be replaced around the same time in the future.
- "The resulting aggregate investment dynamics are surprisingly rich, reflecting the interaction between a replacement cycle, the cross-sectional distribution of the age of the capital stock, and an aggregate shock".

Macro shocks in the auto market also affect the keep vs replace decision

- Firms in the Cooper, *et. al.* model adopt a "buy and keep" strategy since they assumed there is no market for selling used capital.
- Dupor, Li, Saif and Tsai (2024) "The 2008 Auto Market Collapse" also used a buy and hold model of consumer auto purchases to analyze the "intense and violent" 40% collapse in US auto sale during the Great Recession.
- They analyzed three different avenues for the collapse in auto demand: 1) increasing oil prices, 2) falling home values, and 3) falling household income expectations and concluded that the latter was the main cause of the collapse in car sales, i.e. a "precautionary motive" that leads consumers to keep their existing cars longer rather than buy new ones.
- "In response to the shock, households delay replacing existing vehicles, allowing them to smooth the effects of the income shock without significantly adjusting the service flow from their vehicles."

How does the used car market affect shocks to new car demand?

- But in auto markets, consumers have other avenues of substitution: besides keeping their current car, they can switch to having no car, or trade for a used rather than new car when times get tough.
- A shortcoming of the buy and hold assumption is tight link it imposes between the purchase of new cars and the scrappage of old ones.
- In reality, richer consumers are responsible for most purchases of new vehicles and they rarely hold them long enough to be the ones who scrap them. Changes in expected income or risk of unemployment across the business cycle may have little impact on the new car purchase decisions of comparatively rich consumers and scrappage of older cars by poorer consumers may not automatically result in purchases of new ones.
- Poorer consumers may choose to scrap their car and enter the no car state in a recession, so cyclical variation in share of consumers who don't own cars do not necessarily translate into equivalent variations in new car sales.
- Does the used car market dampen the effect of macro shocks on new car sales?

Modeling the impact of macro shocks on the used car market

- Gavazza and Lanteri 2021 Restud "Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets" were one of the first to model how macro shocks affect both new and used markets for cars.
- They argue their model can explain the key facts associated with the collapse in auto sales during the Great Recession through a "chain reaction" effect linking the used car market to the new car market: "households delayed scrapping their (old) cars, thereby decreasing the demand for used cars and depressing their price; in turn, the decline in used-car prices increased the cost of replacing used cars with new ones, thereby reducing the demand for new cars."
- GL find that credit shocks alone are insufficient to explain the 40% drop in new car sales during the Great Recession. When they allow for expectations of a mild income shock that reduces incomes by 2% over a 2 year period, the model "model successfully accounts for a large fraction of the empirical decrease in car sales during the Great Recession."

Limitations of GL's model

- Computational complexity limits the number of used car prices that can be endogenously determined in equilibrium in the GL model. They assume a used car is priced based on its physical condition which evolves stochastically rather than its age which evolves deterministically.
- Their model allows only a single condition of used car so only a single generic 'used car price' is required to clear the used car market.
- Their simulation of the Great Recession assumes that it was caused by an unexpected temporary shock after which economy eventually converges to a stationary equilibrium, but consumers have perfect foresight about the subsequent transition path, which can be calculated by backward induction from the post-shock stationary equilibrium.
- Thus, their simulations of the Great Recession resemble impulse response functions commonly calculated in macroeconomic studies.

Critique of "MIT Shocks"

- Boppart, Krusell and Mitman JEDC 2018 criticized the general approach of analyzing macro shocks as GL and others do as inconsistent with a world where agents expect new shocks to be possible in the future

MIT Shocks

a term coined by Tom Sargent, [that] refers to an unpredictable shock to the steady-state equilibrium of an economy without shocks. That is, in this economy no shocks are expected to ever materialize but nevertheless a shock now occurs. The analysis then focuses on understanding the resulting equilibrium transition along a perfect-foresight path, again under the assumption that no shock will ever occur again. Thus, the described procedure seems hard to square with rational expectations.

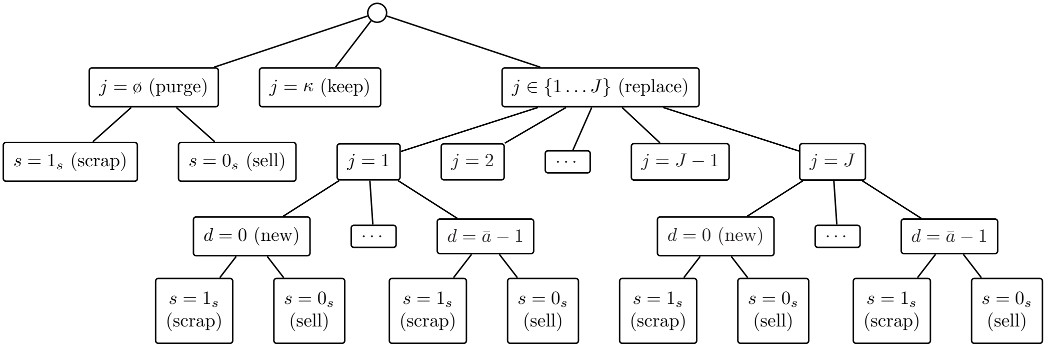
- To our knowledge, we are the first to develop and solve a rational expectations equilibrium model of trading in new and used car markets where agents not only account for the effect of macro shocks on today's prices, but they rationally expect future shocks and how these shocks will affect the market in the future.

Equilibrium trade in automobiles without macro shocks

- We extend the model of equilibrium trade in automobiles of Gillingham, Iskhakov, Munk-Nielsen, Rust and Schjerning (GIMRS) 2022 to allow for persistent macro shocks.
- The GIMRS model has a continuum of atomistic consumers who can own at most one automobile and are subject to idiosyncratic cost/taste shocks, which along with aging and accidents of cars they own, motivate consumers to trade cars, scrapping/selling old cars to purchase a new car, or a used car, or enter the no car state.
- The (idiosyncratic) state of a consumer at time t is (a_t, ε_t) where a_t is the age of their current car (or $s_t = \o$ if they don't own a car), and ε_t is a vector of *IID* taste shocks associated with different car trading decisions. Let d_t be the consumer's decision at time t.
- If $d_t = \kappa$ (keep the current car), utility is $u(a_t) + \epsilon_t(a_t)$.
- If $d_t = \o$ (sell or scrap the current car and not replace it) utility is $max_{s \in \{0,1\}} \Big[u(\o) + \epsilon_t(\o, s) + \mu \big[[p(a_t) - T_s(a)]s + \underline{P}(1-s) \big] \Big]$, where $\underline{P}$ is the scrap value and $p(a)$ is the price of a used car of age a and $T_s(a)$ is a seller-side transaction cost.
- If $d_t = d$ (trade car of age a_t for another of age $d \in \{0, 1, \ldots, \bar{a} - 1\}$), utility is

$$\max_{s \in \{0,1\}} \Big[u(d) + \varepsilon_t(d, s) - \mu \big[p(d) + T_b(d) - [[p(a_t) - T_s(a_t)]s + \underline{P}(1-s)] \big] \Big].$$

Auto choice tree



Car owners' trading problem

$$V(a,\epsilon) = \max \left\{ \begin{array}{l} v(a,\kappa) + \epsilon(\kappa); \\ \max_{s\in\{1_s,0_s\}} \left[v(a,\emptyset,s) + \epsilon(\emptyset,s) \right]; \\ \max_{\substack{d\in\{0,1,\ldots,\bar{a}-1\},\\ s\in\{1_s,0_s\}}} \left[v(a,d,s) + \epsilon(d,s) \right] \end{array} \right\}$$

States Choices

- Existing car (i, a), traded car (j, d)
- When existing car (i, a) is replaced, there is additional scrappage choice $s \in \{0_s, 1_s\}$: to sell or to scrap the replaced car.
- Similar recursive maximization problems for consumers with no car and owner of car of terminal age $\bar{a}$

Notation: $p \in R^{\bar{a}-1}$ is the vector of used car prices of ages $a \in \{1, 2, \ldots, \bar{a}-1\}$ (cars of age $\bar{a}$ are "clunkers" and cannot be driven and have only scrap value $\underline{P}$). So $p(a)$ is the price of a used car of age a. We extend p to include new car $p(0) = \overline{P}$ and scrap value for clunker, $p(\bar{a}) = \underline{P}$, so the extended $p \in R^{\bar{a}+1}$.

Choice-specific value functions

$$
\begin{aligned}
v(a, \text{ø}, 1_s, p) &= u(\text{ø}) + \mu \underline{P} + \beta EV(\text{ø}) \\
v(a, \text{ø}, 0_s, p) &= u(\text{ø}) + \mu[p(a) - T_s(p, a)] + \beta EV(\text{ø}) \\
v(a, \kappa, p) &= u(a) + \beta(1-\alpha) EV(a+1) + \beta\alpha EV(\bar{a}) \\
v(a, d, 1_s, p) &= u(d) - \mu[p(d) - \underline{P} + T_b(p, d)] + \\
&\quad + \beta(1-\alpha) EV(d+1) + \beta\alpha EV(\bar{a}) \\
v(a, d, 0_s, p) &= u(d) - \mu[p(d) - p(a) + T_s(p, a) + T_b(p, d)] + \\
&\quad + \beta(1-\alpha) EV(d+1) + \beta\alpha EV(\bar{a})
\end{aligned}
$$

States Choices $\rightarrow$ Current period utility Future value

- Similar expressions for consumers with no car and owner of car of terminal age $\bar{a}$
- Note that v is an explicit function of car prices p.

Solving the consumers' problem

$$EV(a) = \sigma \log \left\{ \sum_{d,s} \exp \left[\frac{v(a,d,s,p)}{\sigma} \right] \right\}$$

- **Fixed point of Bellman operator in EV space**

$$EV(p) = \Gamma\big(EV(p), p\big)$$

- Conditional choice probabilities are then analytical, similar to

$$\Pi(d,s|a,p) = \frac{\exp\left[v(a,d,s,p)/\sigma\right]}{\sum_{d',s'} \exp\left[v(a,d',s',p)/\sigma\right]}.$$

  - *Note:* EV is an implicit function of p. Since v depends on p, the CCPs also depend on p.
  - Fixed point solved using gradient-based Newton method with very precise starting values

DNFXP algorithm

- **Outer optimization:** Maximum likelihood search over θ
- **Inner equilibrium solver:** Find prices, $p^\star$, so $ED(p^\star, q(p^\star)) = 0$
- **Excess demand:** Each trial value of p requires
  1. Solve single agent DP/fixed point given p
  2. Compute transition matrices $\Omega(P)$ and Q
  3. Find stationary holdings distribution $q(p) : q = q\Omega(p)Q$
  4. Evaluate excess demand $ED(p, q(p))$.

Stationary flow market equilibrium framework

Assumptions

1. Infinitely inelastic supply of new cars $\overline{P}$
2. Infinitely elastic demand for scrapped cars $\underline{P}$
3. $(\bar{a}-1)\times 1$ vector of endogenously determined used car prices p

Definition: Ownership Distribution q

$$q = \Big(\underbrace{q_1}_{\text{age } 1}, \ldots, \underbrace{q_{\bar{a}}}_{\text{car } \bar{a}}, \underbrace{q_{\o}}_{\text{no car}}\Big) \in \mathbb{R}^{\bar{a}+1}$$

- q_a is the fraction of consumers holding car of age a, so $q \in \Delta(\bar{a})$, the $\bar{a}$-dimensional simplex.
- By our timing assumption new cars purchased in any time period are accounted for as one-years-old cars in the next time period, so $q_0 = 0$ always!

Stationary Equilibrium (SE)

Definition: Stationary Equilibrium (SE)

A pair $q^\star \in \mathbb{R}^{\bar{a}+1}$ and $p^\star \in \mathbb{R}^{\bar{a}-1}$ such that

1. Consumers maximize expected discounted utility,
2. Secondary market clears for all tradeable cars,
3. Ownership distribution is time-invariant.

The dynamics of the ownership distribution q are described by

- *Trade* transition probability matrix $\Omega(p)$ composed of conditional choice probabilities of trading decisions
- *Physical* transition probability matrix Q: ageing of cars + stochastic transitions to terminal age $\bar{a}$ (involuntary scrappage)

A SE will also be in *flow equilibrium* each period, fraction of households scrapping cars equals the fraction of households buying new cars.

Trade transition probability matrix $\Omega(p)$

$\Omega(p) = (\bar{a}+1) \times (\bar{a}+1)$ matrix given by

$$\Omega(p) = \begin{bmatrix} \Pi(1|1) + \Pi(\kappa|1) & \dots & \Pi(\bar{a}-1|1) & \Pi(0|1) & \Pi(\emptyset|1) \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \Pi(1|\bar{a}-1) & \dots & \Pi(\bar{a}-1|\bar{a}-1) + \Pi(\kappa|\bar{a}-1) & \Pi(0|\bar{a}-1) & \Pi(\emptyset|\bar{a}-1) \\ \Pi(1|\bar{a}) & \dots & \Pi(\bar{a}-1|\bar{a}) & \Pi(0|\bar{a}) & \Pi(\emptyset|\bar{a}) \\ \Pi(1|\emptyset) & \dots & \Pi(\bar{a}-1|\emptyset) & \Pi(0|\emptyset) & \Pi(\emptyset|\emptyset) \end{bmatrix}$$

$q \cdot \Omega(p)$ is the distribution of cars *immediately after the trading phase*

- $\Pi(d|a,p)$ is the probability of trading car of age a for car of age d
- $\Pi(a|a,p) + \Pi(\kappa|a,p)$ is probability of transitioning $a \to a$
- $\Pi(\emptyset|a,p)$ is probability of going to no car state
- $\Pi(\emptyset|\emptyset,p)$ is probability of remaining in the no car state
- Next to last column of $\Omega(p)$ is the CCP for purchasing a new car

Aging/accident transition probability matrix Q

$Q = (\bar{a}+1) \times (\bar{a}+1)$ matrix given by

$$Q = \begin{bmatrix} 0 & 1-\alpha & \dots & 0 & \alpha \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1-\alpha & \alpha \\ 0 & 0 & \dots & 0 & 1 \\ 1-\alpha & 0 & \dots & 0 & \alpha \end{bmatrix}$$

- Car becomes 1 year older if not totalled in an accident, with probability $1-\alpha$
- Total loss due to accident with probability α

$q \cdot \Omega(p)Q$ is *ownership distribution* in the *next period*

The stationary holdings distribution

$$\underbrace{q}_{t} \rightarrow \underbrace{q\Omega(p)}_{\text{after trading}} \rightarrow \underbrace{q\Omega(p)Q}_{t+1}$$

Condition for time invariance of the ownership distribution:

$$q = q\Omega(p)Q$$

Theorem (Uniqueness of stationary ownership distribution)

If scale of GEV shocks disctribution is positive then stationary ownership distribution is unique.

Excess demand functions

- **Demand:** Fraction of consumers buying a given car d:

$$D_d(P, q) = \Pi(d|\o, p)q_{\o} + \sum_{a=1}^{\bar{a}} \Pi(d|a, p)q_a$$

- **Supply:** Fraction of owners that sell (not scrap) their car d

$$S_d(p, q) = \big(1 - \Pi(\kappa|d, p)\big)\big(1 - \Pi(1_s|d, p)\big)q_d$$

- **Market clearing condition** is the non-linear system of equations in ownership shares q and prices p

$$ED(p, q) \equiv D(p, q) - S(p, q) = 0$$

  - The stationarity condition $q = q(p)$ implies that $q(p)$ is an implicit function of p.
  - Thus, equilibrium reduces to solving the $\bar{a} - 1$ equations $\text{ED}(p, q(p)) = 0$ for the $(\bar{a} - 1)$ unknowns p.

Existence of stationary equilibrium

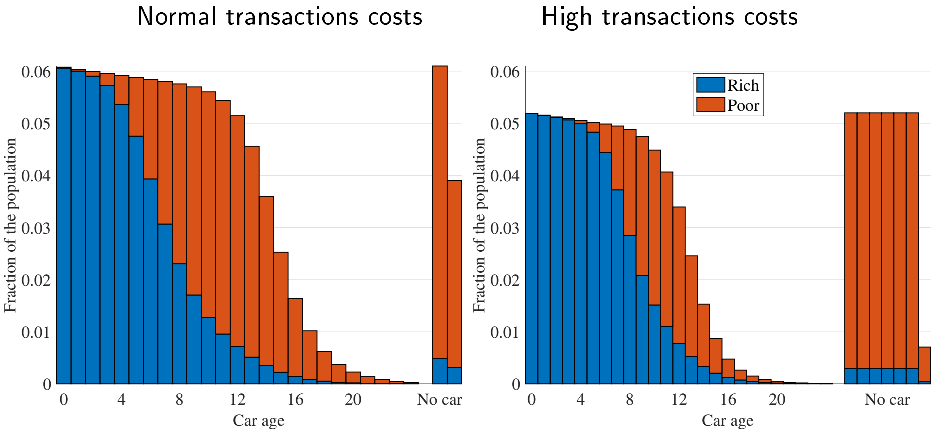
Theorem (Equilibrium existence)

The stationary equilibrium for the automobile economy with the idiosyncratically heterogeneous consumers $(q^\star, p^\star)$ exists, and in equilibrium we have:

$$q^\star = q^\star \Omega(p^\star) Q,$$
$$0 = ED(p^\star, q^\star).$$

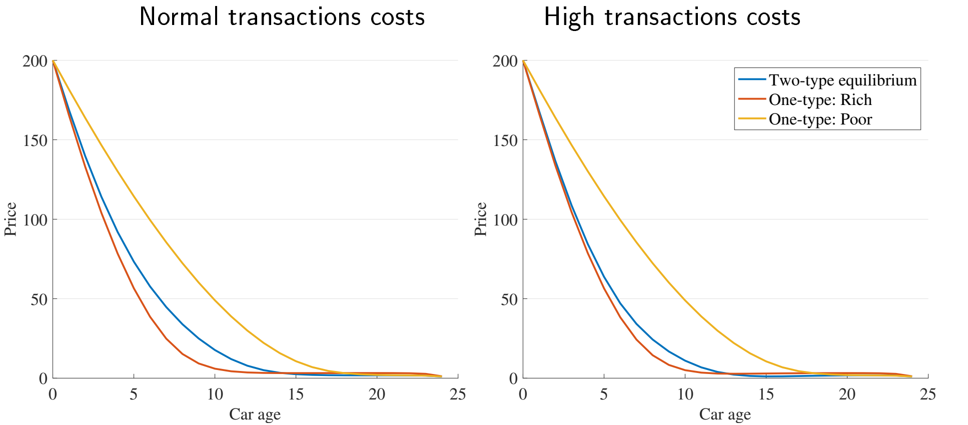
- We prove $q^\star$ is unique given $p^\star$, but have not been able to prove that $p^\star$ is unique, i.e. there might be multiple solutions $p^\star$ to $\mathrm{ED}(p, q(p)) = 0$.
- However, have not seen any signs of multiplicity in computational practice

Illustrative example: ownership by two consumer types



- Sorting of consumers in each regime
- Heterogeneous effects of transaction costs

Illustrative example: equilibrium prices - two consumer types



High transactions costs:

- Equilibrium prices similar to economy where all consumers are rich (many poor consumers are now driven out of the market)
- Transactions costs limits gains from trade

Adding Macro Shocks to the Model

- Note the quasi-linear preference specification of the GIMRS model, standard to discrete choice models used in IO. How does the model capture "income/wealth/liquidity effects"?
- A key finding of GIMRS's empirical analysis is that a key parameter is μ, *the marginal utility of money,* governing consumer price sensitivity.
- Rich consumers have low μ: they are not as price-sensitive and this makes them more likely to buy new cars
- Poor consumers have high μ: they are price-sensitive and this makes them more likely either not to have a car, or to buy used cars instead of new ones
- This motivates a simple way to model macro shocks. Assume all consumers have same μ but μ depends on a binary macro state $m \in \{0, 1\}$, and $\mu(0) > \mu(1)$.
- $m = 0$ **recession state** high marginal utility of money in this state *discourages* consumers from buying new cars during recessions.
- $m = 1$ **boom state** low marginal utility of money in this state *encourages* consumers to buy new cars in a boom period.

Rational expectations with macro states

- Assume $\{m_t\}$ evolves as an exogenous Markov chain with 2×2 transition probability matrix M with elements $M_{m,m'}$ equal to the transition probability $M_{m,m'} = \text{Prob}(m_{t+1} = m' | m_t = m)$.
- Now must extend consumer's DP problem to include m_t as a *macro state variable*
- But is m_t enough for consumers to rationally forecast future prices and quantities?
- **NO!** Equilibrium prices depend on q but with macro shocks, quantities will evolve over time, i.e. as an endogenously determined equilibrium outcome $\{q_t\}$ driven by the exogenous macro "forcing process" $\{m_t\}$.
- Thus, consumers need to keep track of both $\{m_t, q_t\}$. Is this process Markovian?
- **YES!** We will show that $\{m_t, q_t\}$ is jointly Markovian, with *stationary transition probability* $f(m', q' | m, q)$.
- In a rational expectations equilibrium, prices that clear the market will be a function of (m_t, q_t), i.e. excess demand will be of the form $\text{ED}(p, q_t, m_t)$ so market clearing prices will solve $\text{ED}(p(q_t, m_t), q_t, m_t) = 0$, implying $p(q, m)$ is an implicit function of the *macro state* (q, m). This is similar to the *Krusell-Smith problem,* we need to keep track of an endogenous macro state/distribution, i.e. q_t rather than the distribution of wealth $F_t(w)$.

Individual DP problem under RE

- If consumers have rational expectations, they need to use all the available information to predict future macro states, (q_t, m_t) which in turn helps them predict future car prices $p(q_t, m_t)$. This means they need to know:
  1. M the transition probability matrix for $\{m_t\}$
  2. a belief about how q_{t+1} is determined given (q_t, m_t), i.e. a *belief* of the form $q' = \phi(q, m)$
  3. how prices p_t are determined given (q_t, m_t), i.e. a *price function* of the form $p = \rho(q, m)$.
- In a *rational expectations equilibrium* (REE or RE for short), these beliefs must be rational, i.e. correct.
- Let $V(a, \varepsilon, m, q, \rho, \phi)$ denote the solution to the consumers's DP/car trading problem that depends on the *idiosyncratic, consumer-specific states variables* (a, ε) and the *common macro state variables* (q, m), as well as consumer "belief functions" (ρ, ϕ).
- Since v depends on (m, q, ρ, ϕ) it follows that the CCPs also depend on them too.
- Since the trade transition probability Ω is constructed from CCPs, it follows that we can write $\Omega(q, m, \rho, \phi)$

Objective or true law of motion for q

- In RE, the macro shocks imply that car holdings q is not time-invariant. But it still evolves according to the same equation given previously in deriving the condition for a stationary holdings distribution in SE

$$q_{t+1} = q_t \Omega(q_t, m_t, \rho(q_t, m_t), \phi(q_t, m_t)) Q, \quad \forall (q_t, m_t).$$

- Further, markets need to clear, so there is a price function $p(q_t, m_t)$ satisfying

$$0 = \mathrm{ED}(q_t, m_t, \rho(q_t, m_t), \phi(q_t, m_t)), \quad \forall (q_t, m_t).$$

- Bellman's equation holds given beliefs above

$$EV(a, q, m, \rho, \phi) = \Gamma(EV(a, q, m, \rho, \phi), a, q, m, \rho, \phi),$$

so v and the CCPs and therefore Ω and ED depend not only on (q, m) but also on consumers' beliefs (ρ, ϕ).

Rational Expectations Equilibrium (RE)

Definition: Rational Expectations Equilibrium (RE)

A pair of functions $\rho^{\star}(q,m) : \mathbb{R}^{2(\bar{a}+1)} \to \mathbb{R}^{\bar{a}-1}$ and $\rho^{\star}(q,m) : \mathbb{R}^{2(\bar{a}+1)} \to \mathbb{R}^{\bar{a}+1}$ such that

1. Consumers maximize expected discounted utility taking into account the macro state (q,m) with beliefs given by (ρ,ϕ).
2. Consumers have correct beliefs about the prices $p=\rho(q,m)$ that clear secondary markets for all (q,m).
3. Consumers have correct beliefs about the evolution of the holdings distribution $q'=\phi(q,m)$.

This reduces to a system of functional equations that hold for all (q,m), $m \in \{0,1\}$, $q \in \Delta(\bar{a})$

$$\begin{aligned} \phi(q,m) &= q\Omega(q,m,\rho(q,m),\phi(q,m))Q \\ 0 &= \mathrm{ED}(q,m,\rho(q,m),\phi(q,m)) \end{aligned}$$

and Bellman's equation holds for all (q,m) with beliefs given by (ρ,ϕ) above.

Toy model solution $\bar{a} = 2$

Parameters	**Values**
$(u_0, u_1, u_\emptyset)$	$(10, 5, 0)$
(T^s, T^b)	$(0, 1.5)$
$(\overline{P}, \underline{P})$	$(100, 10)$
$\mu(m)$	$(0.10, 0.15)$
$M(1\|1), M(0\|0)$	$(0.75, 0.75)$

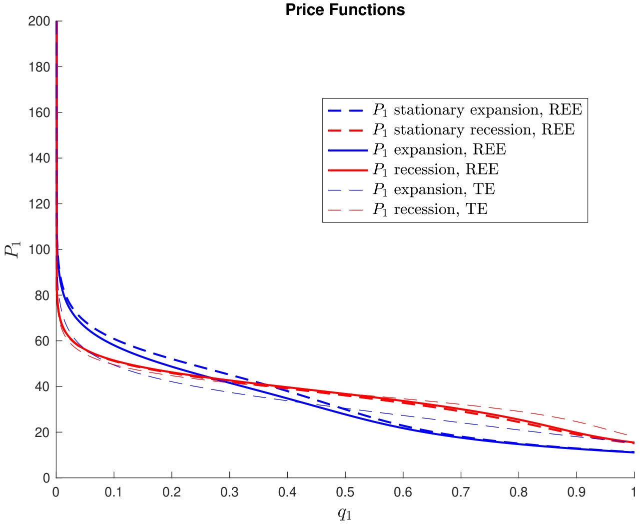
Table: Toy Model parameters

- **Observation:** The future looks the same for consumers in state $\bar{a}$ as in state $\emptyset$. This implies what we only need to track two state variables, $q_{\bar{a}} + q_\emptyset$ and q_1. Furthermore, since

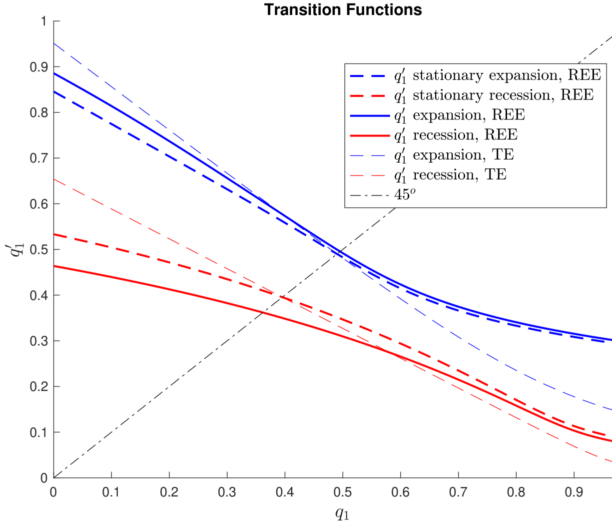
$$q_\emptyset + q_{\bar{a}} = 1 - q_1$$

it follows that the "state" becomes essentially 1 dimensional even though $q \in R^3$, i.e. the 2-dimension simplex.

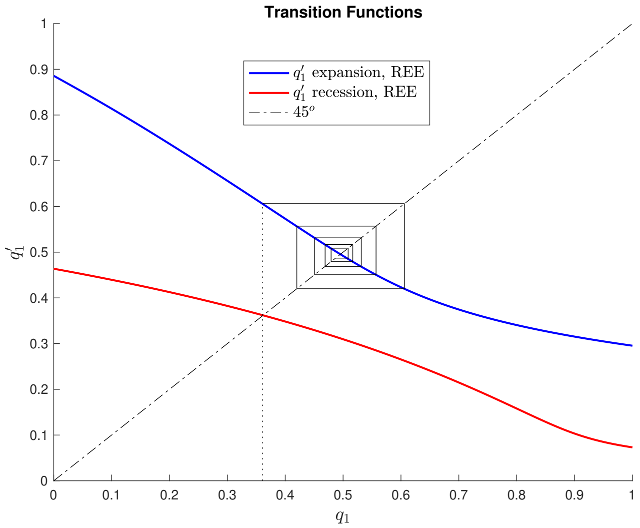
Equilibrium price functions $p(q, m)$



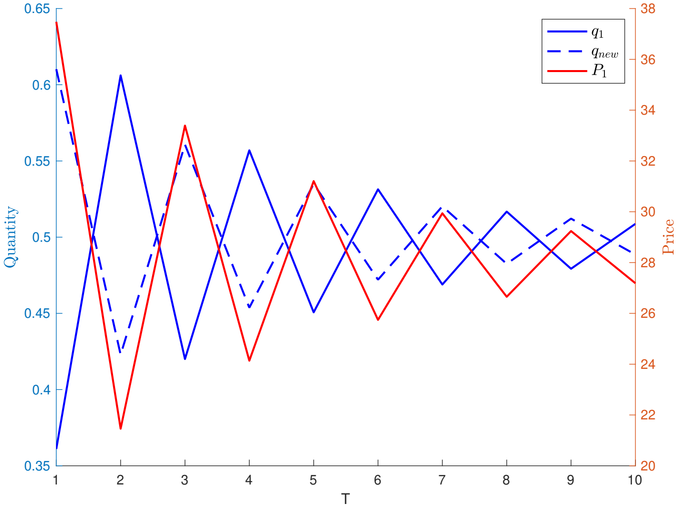
State transition functions $\phi(q, m)$



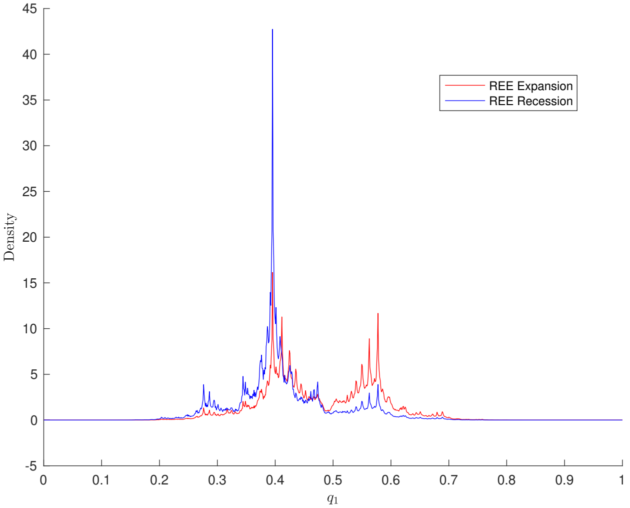
Cobweb cycles from a one time transition $0 \to 1$ (recession to boom)



Impulse response from a one time transition $0 \to 1$ (recession to boom)



Ergodic distribution of q, given boom and bust



Calibrated model of Danish economy with $\bar{a} = 13$

Assume a quadratic utility function in car ages

$$u(a) = u_0 + u_1 a + u_2 a^2$$

Parameters	Values
(u_0, u_1, u_2)	$(6.250637, -1.194026, 0.0564)$
u_n	0
switching cost	3.844278
switching cost n	$3.844278 + 1.482223$
$(\overline{P}, \underline{P})$	$(199.81, 7.11)$
$\mu(m)$	$(0.093, 0.096)$
$M(0\|0), M(1\|1)$	$(0.75, 0.75)$

Table: Model parameters

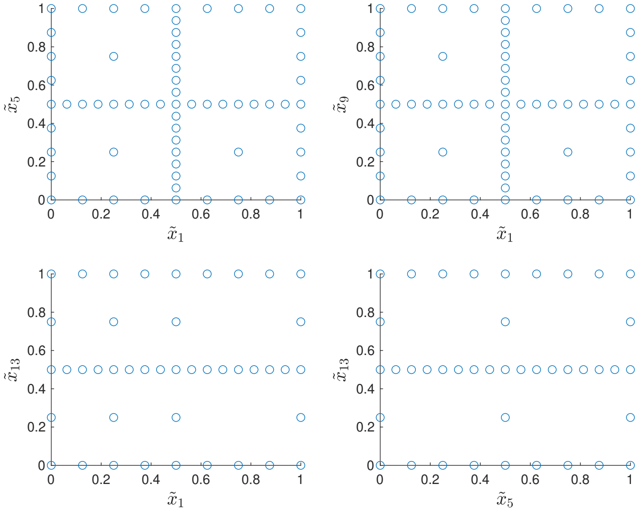
Solve model on an adaptive sparse grid

- Model solved using the GDSGE toolbox of Cao, Luo and Nie (2023) *Review of Economic Dynamics* "Global DSGE Models"
- Implement Adaptive Sparse Grid of Brumm and Scheidegger (2017) *Econometrica* "Using Adaptive Sparse Grids to Solve High Dimensional Dynamic Models"
- Used the following transformations to transform state space from simplex to a rectangular grid to apply the ASP algorithm

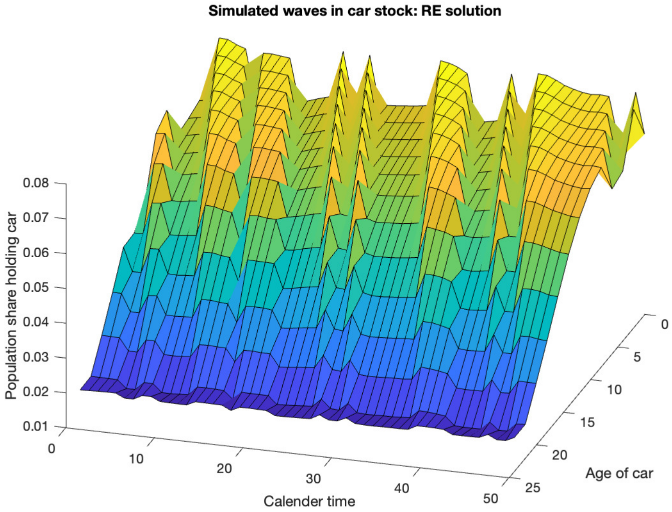
$$
\begin{aligned}
q_1 &= x_1 \\
q_2 &= (1 - q_1)x_2 \\
q_3 &= (1 - q_1 - q_2)x_3 \\
&\vdots \\
q_{13} &= (1 - q_1 - q_2 - \cdots - q_{12})x_{13}
\end{aligned}
$$

where $0 < \underline{x}_i \leq x_i \leq \bar{x}_i < 1, \forall i = 1, 2, \ldots, 13$, where $\underline{x}_i, \bar{x}_i$ are exogenous bounds.

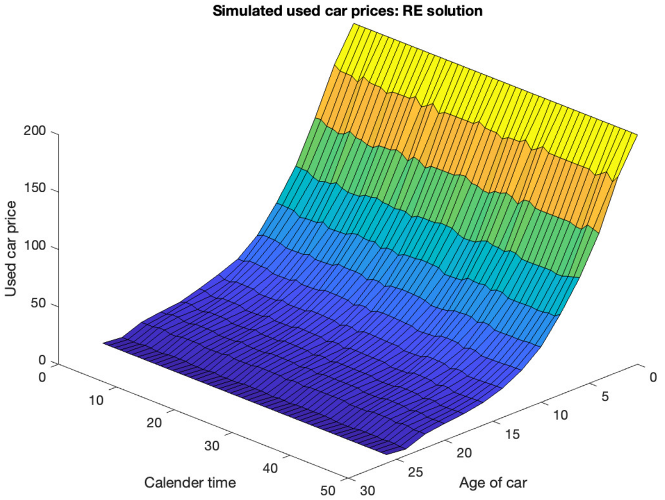
Adaptive sparse grids used to solve for RE



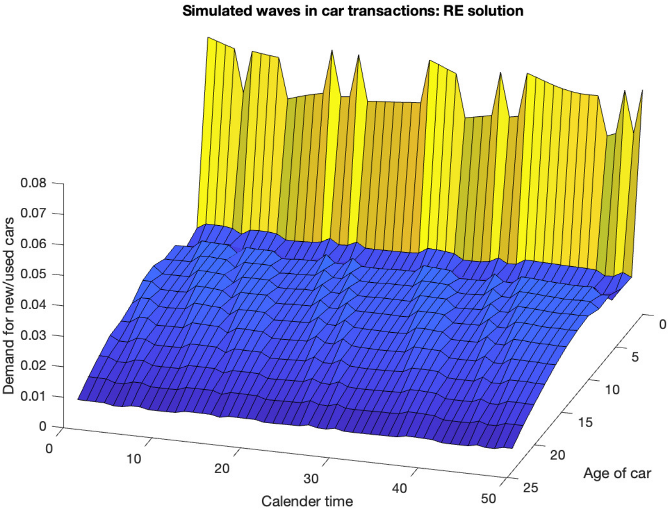
Waves in q from 50 period simulation of RE



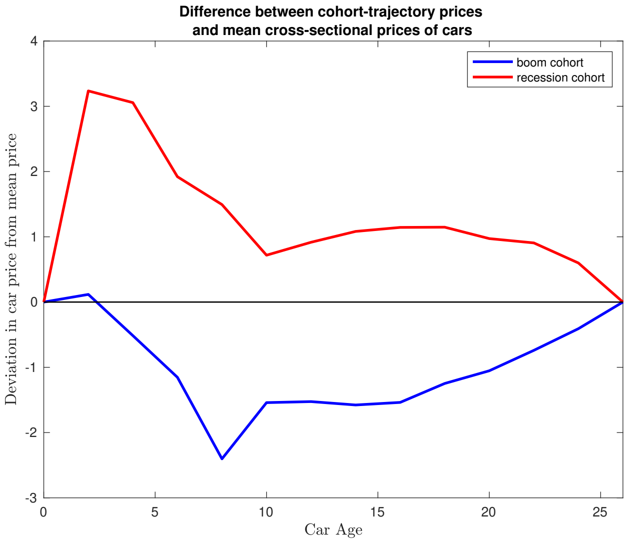
Waves in p from 50 period simulation of RE



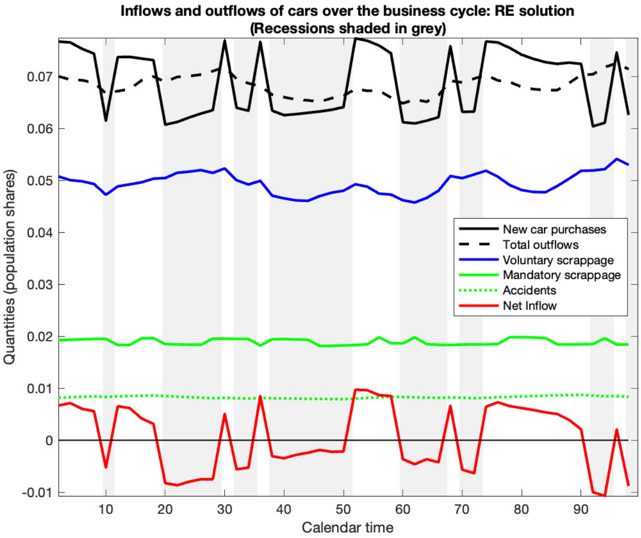
Waves in car transactions from 50 period simulation of RE



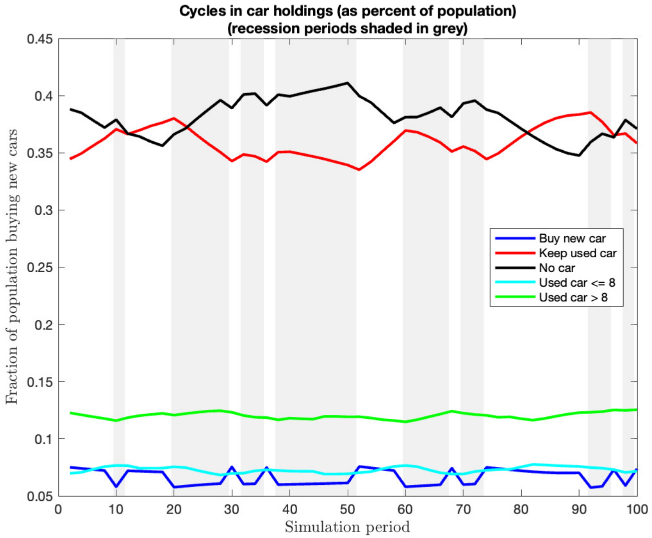
Cohort effects of recessions/booms on car prices



Inflows and outflows of cars from the economy, RE



Main effect: cyclical variation in keeping and transitions to no car state



Critique of RE

- RE entails a severe curse of dimensionality at multiple levels.
- If there are J different car types, then RE requires individuals to track the holdings distributions of all J car types, $(q_{t1}, q_{t2}, \dots, q_{tJ})$, so the dimensionality of the consumer problem scales linearly with number of types.
- But even using adaptive sparse grids, time to solve a DP with K continuous state variables scales exponentially in K.
- If there are N persistent types $(\tau_1, \dots, \tau_N)$ in the economy with different holdings distributions, $q_t^{\tau_i}$, $i = 1, \dots, N$ then RE requires each agent to track the entire vector of holdings distributions $\{q_t^{\tau_1}, \dots, q_t^{\tau_N})$ so solution time scales exponentially in N as well as J.

Benjamin Moll (2024) critique of RE solution concept

"It is self-evident that real-world households and firms do not forecast prices by forecasting distributions and instead solve simpler problems." from *Economic Journal* (forthcoming) "The Trouble with Rational Expectations in Heterogeneous Agent Models: A Challenge for Macroeconomics"

Simpler alternatives to RE

- Examples of simpler problems is the "Krusell-Smith" approximation: instead of trying to forecast q_t focus on a reduced DP problem that only attempts to forecast moments of q_t, e.g.

$$E\{a|q_t\} = \sum_{a=1}^{\bar{a}} a\hat{q}_t(a)$$

  where $\hat{q}_t$ is the conditional distribution of car ownership, excluding the no car state, $q_t(\emptyset)$.
- We compare an even simpler notion of equilibrium in the spirit of Moll's critique, *Temporary Equilibrium* (TE)
- Jean-Michel Grandmont (1977) *Econometrica* "Temporary General Equilibrium Theory"
- TE is a generalization of RE that includes RE as a special case, allowing essentially arbitrary forecasting rules for uncertain future payoff-relevant states. In the auto market, $\{p_t\}$ is the uncertain payoff relevant state.
- Our implementation of TE is especially naive, using a very simple forecasting rule for future prices: *consumers believe future prices are given by $p(m)$, the SE price vector assuming that future macro states m_{t+s}, $s > 0$ will forever remain at the current macro state, i.e. m_t.*

Temporary Equilibrium (TE)

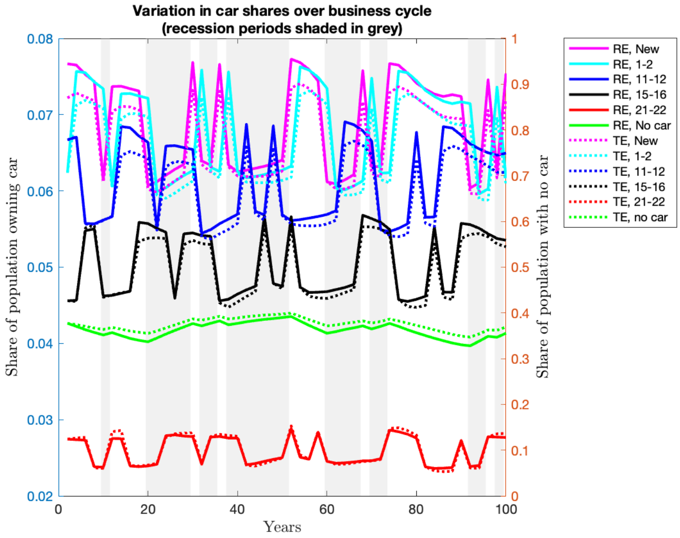
Definition: Temporary Equilibrium (TE)

A sequence $\{p_t, q_t\}$ that satisfies

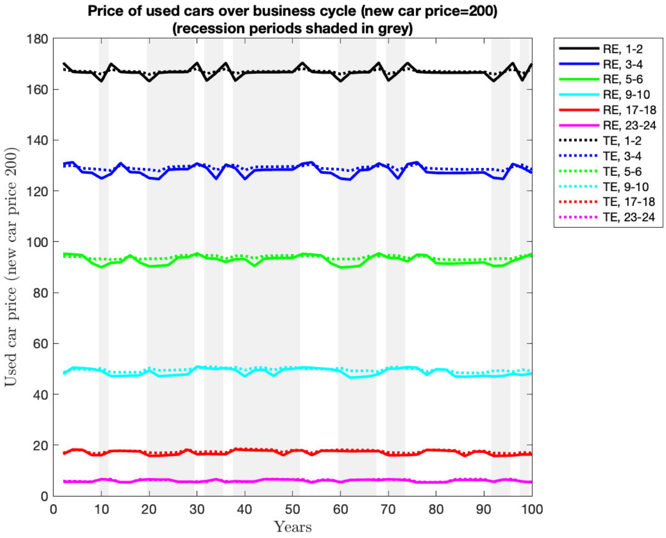
1. Each period t, consumers maximize expected discounted utility taking current market clearing price p_t as given but expecting that future prices are $p(m_t)$, the SE values assuming the economy forever remains in the current macro state m_t.
2. The secondary market clears for all tradeable cars, $\text{ED}(p_t, q_t, p(m_t)) = 0$,
3. The ownership distribution evolves according to $q_{t+1} = q_t \Omega(p_t, p(m_t)) Q$.

- Thus, consumers do not need to pay attention to q_t in solving their DP problems, they only need to be able to to compute $p(m_t)$, the SE corresponding to the current macro state m_t.
- As a result, it is much faster to calculate TE solutions or "trajectories" $\{p_t, q_t\}$ because of the assumed myopia of consumers when it comes to forecasting future car prices.

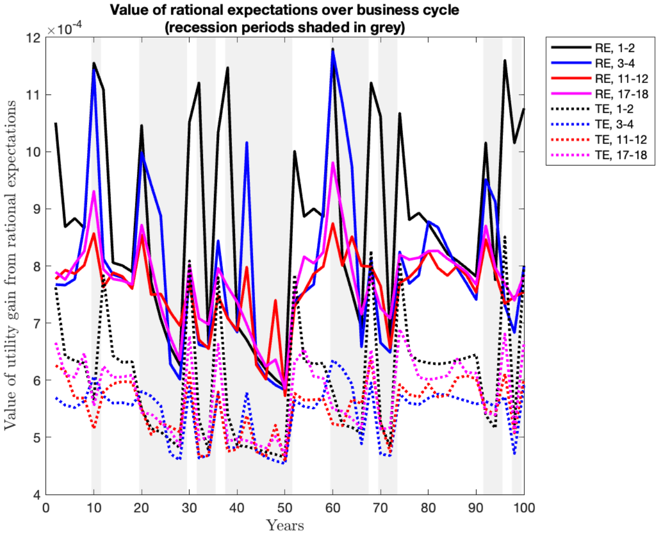
Evolution of Quantities, RE vs TE



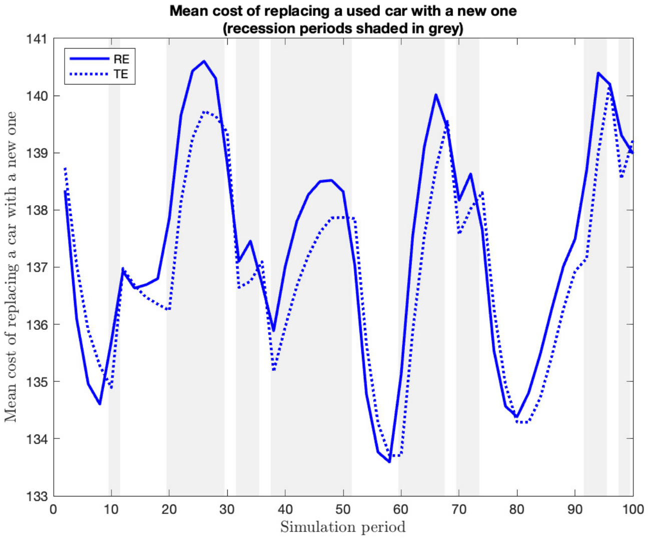
Evolution of Prices, RE vs TE



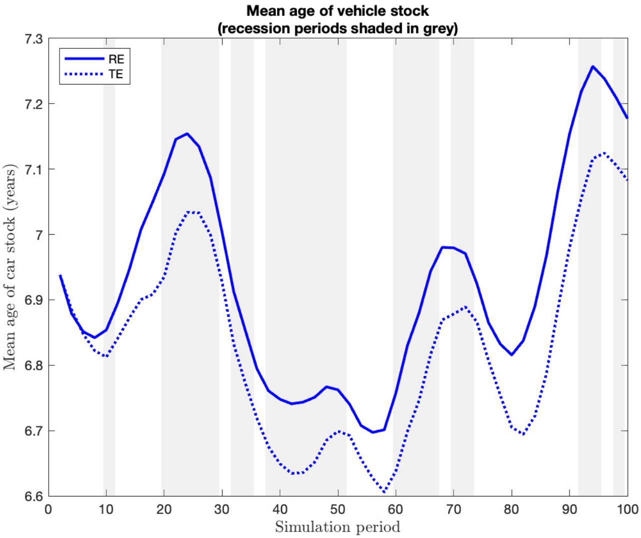
Small gain in welfare to being sophisticated



Mean cost of replacing a used car with a new one, RE vs TE



Mean age of the vehicle stock, RE vs TE



Conclusion

- Macro shocks have well known impacts on major consumer durable purchases such as cars, inducing intertemporal substitution during recession periods that create persistent cycles or *waves* in the age distribution of the used car stock.
- We generated *equilibrium waves* by extending the stationary equilibrium concept of Gillingham *et. al.* 2022 that rules out the effects of macro shocks on prices and quantities of cars. We analyzed two different equilibrium concepts, *rational expectations equilibrium* and *temporary equilibrium*, solved and simulated with our model calibrated to Danish data.
- The two equilibrium concepts generate realistic waves and very similar cyclical patterns, though TE is vastly easier to compute than RE.
- We conjecture the similarity in outcomes is a consequence of high *subtitutabiity in the new and used car market* thar implies that only small price adjustments are needed to clear the market in response to much more substantial quantity shocks. As a result, *loss of consumer welfare from the naive TE approach is inconsequential.*
- The model reveals that the main cyclical impact and margins of adjustments to macro shocks is not so much in new car sales as it is in variations in the propensity to *keep your current car rather than buy a new one, or trade your car and enter the no car state.*